OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE **4733/01**

MATHEMATICS

Probability & Statistics 2

TUESDAY 15 JANUARY 2008 Morning

Time: 1 hour 30 minutes

Additional materials: Answer Booklet (8 pages)
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of **4** printed pages.

1 The random variable T is normally distributed with mean μ and standard deviation σ. It is given that $P(T > 80) = 0.05$ and $P(T > 50) = 0.75$. Find the values of μ and σ. [6]

2 A village has a population of 600 people. A sample of 12 people is obtained as follows. A list of all 600 people is obtained and a three-digit number, between 001 and 600 inclusive, is allocated to each name in alphabetical order. Twelve three-digit random numbers, between 001 and 600 inclusive, are obtained and the people whose names correspond to those numbers are chosen.

(i) Find the probability that all 12 of the numbers chosen are 500 or less. [3]

(ii) When the selection has been made, it is found that all of the numbers chosen are 500 or less. One of the people in the village says, "The sampling method must have been biased." Comment on this statement. [2]

3 The random variable G has the distribution $Po(\lambda)$. A test is carried out of the null hypothesis $H_0 : \lambda = 4.5$ against the alternative hypothesis $H_1 : \lambda \neq 4.5$, based on a single observation of G. The critical region for the test is $G \leqslant 1$ and $G \geqslant 9$.

(i) Find the significance level of the test. [5]

(ii) Given that $\lambda = 5.5$, calculate the probability that the test results in a Type II error. [3]

4 The random variable Y has the distribution $N(\mu, \sigma^2)$. The results of 40 independent observations of Y are summarised by

$$\Sigma y = 3296.0, \quad \Sigma y^2 = 286\,800.40.$$

(i) Calculate unbiased estimates of μ and σ^2. [4]

(ii) Use your answers to part **(i)** to estimate the probability that a single random observation of Y will be less than 60.0. [2]

(iii) Explain whether it is necessary to know that Y is normally distributed in answering part **(i)** of this question. [1]

5 Over a long period the number of visitors per week to a stately home was known to have the distribution $N(500, 100^2)$. After higher car parking charges were introduced, a sample of four randomly chosen weeks gave a mean number of visitors per week of 435. You should assume that the number of visitors per week is still normally distributed with variance 100^2.

(i) Test, at the 10% significance level, whether there is evidence that the mean number of visitors per week has fallen. [7]

(ii) Explain why it is necessary to assume that the distribution of the number of visitors per week (after the introduction of higher charges) is normal in order to carry out the test. [2]

6 The number of house sales per week handled by an estate agent is modelled by the distribution Po(3).

(i) Find the probability that, in one randomly chosen week, the number of sales handled is

(a) greater than 4, [2]

(b) exactly 4. [2]

(ii) Use a suitable approximation to the Poisson distribution to find the probability that, in a year consisting of 50 working weeks, the estate agent handles more than 165 house sales. [5]

(iii) One of the conditions needed for the use of a Poisson model to be valid is that house sales are independent of one another.

(a) Explain, in non-technical language, what you understand by this condition. [1]

(b) State another condition that is needed. [1]

7 A continuous random variable X_1 has probability density function given by

$$f(x) = \begin{cases} kx & 0 \leqslant x \leqslant 2, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(i) Show that $k = \frac{1}{2}$. [2]

(ii) Sketch the graph of $y = f(x)$. [2]

(iii) Find $E(X_1)$ and $Var(X_1)$. [5]

(iv) Sketch the graph of $y = f(x - 1)$. [2]

(v) The continuous random variable X_2 has probability density function $f(x - 1)$ for all x. Write down the values of $E(X_2)$ and $Var(X_2)$. [2]

8 Consultations are taking place as to whether a site currently in use as a car park should be developed as a shopping mall. An agency acting on behalf of a firm of developers claims that at least 65% of the local population are in favour of the development. In a survey of a random sample of 12 members of the local population, 6 are in favour of the development.

(i) Carry out a test, at the 10% significance level, to determine whether the result of the survey is consistent with the claim of the agency. [7]

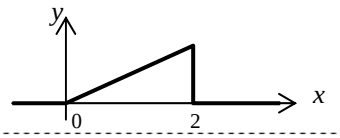
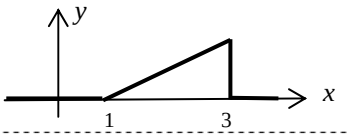
(ii) A local residents' group claims that no more than 35% of the local population are in favour of the development. Without further calculations, state with a reason what can be said about the claim of the local residents' group. [2]

(iii) A test is carried out, at the 15% significance level, of the agency's claim. The test is based on a random sample of size $2n$, and exactly n of the sample are in favour of the development. Find the smallest possible value of n for which the outcome of the test is to reject the agency's claim. [4]

Final Mark Scheme (post-standardisation)

1		$\frac{80-\mu}{\sigma} = \Phi^{-1}(0.95) = 1.645$ $\frac{\mu-50}{\sigma} = \Phi^{-1}(0.75) = 0.674(5)$ Solve simultaneously $\mu = 58.7$, $\sigma = 12.9$	M1 B1 A1 M1 A1 A1	**6**	Standardise once with Φ^{-1}, allow σ^2, cc Both 1.645 (1.64, 1.65) and [0.674, 0.675], ignore signs Both equations correct apart from wrong *z*, *not* 1–1.645 Solve two standardised equations μ, a.r.t 58.7 σ, a.r.t. 12.9 [*not* σ^2] [σ^2: M1B1A0M1A1A0]
2	(i)	Let *R* denote the number of choices which are 500 or less. $R \sim B(12, \frac{5}{6})$ $P(R = 12) = (\frac{5}{6})^{12}$ [=0.11216] = **0.112**	M1 M1 A1	**3**	B(12, $\frac{5}{6}$) stated or implied, allow 501/600 etc p^{12} or q^{12} or equivalent Answer, a.r.t. 0.112 [SR: $\frac{500}{600} \times \frac{499}{599} \times \frac{498}{598} \times \ldots$; 0.110: M1A1] [M1 for 0.910 or 0.1321 or vague number of terms]
	(ii)	Method unbiased; unrepresentative by chance	B1 B1	**2**	State that method is unbiased Appropriate comment (e.g. "not unlikely") [SR: partial answer, e.g. not necessarily biased: B1]
3	(i)	P(≤ 1) = 0.0611 P(≥ 9) = 1 – P(≤ 8) = 1 – 0.9597 = 0.0403 0.0611 + 0.0403 [= 0.1014] = **10.1**%	B1 M1 A1 M1 A1	**5**	0.0611 seen Find P(≥ 9), allow 8 or 10 [0.0866, 0.0171] 0.0403 correct Add probabilities of tails, *or* 1 tail × 2 Answer [10.1, 10.2]% or probability
	(ii)	$P(2 \le G \le 8)$ = 0.8944 – 0.0266 [= 0.8678] = **0.868**	M1 M1 A1	**3**	Attempt at $P(2 \le G \le 8)$, *not* isw, allow $1 \le G \le 9$ etc Po(5.5) tables, P(≤ top end) – P(≤ bottom end) Answer, a.r.t. 0.868, allow %
4	(i)	$\hat{\mu} = \bar{y} = \frac{3296.0}{40} = 82.4$ $\frac{286800.4}{40} - 82.4^2$ [= 380.25] $S^2 \times \frac{40}{39}$; = 390	B1 M1 M1 A1	**4**	Mean 82.4, c.a.o. Use correct formula for biased estimate Multiply by $n/(n-1)$ [SR: all in one, M2 or M0] Variance 390, c.a.o.
	(ii)	$\Phi\left(\frac{60-82.4}{\sqrt{390}}\right) = \Phi(-1.134)$ = 1 – 0.8716 = **0.128**	M1 A1	**2**	Standardise, allow 390, cc or biased estimate, +/–, do not allow $\sqrt{n}$ Answer in range [0.128, 0.129]
	(iii)	No, distribution irrelevant	B1	**1**	"No" stated or implied, any valid comment
5	(i)	$H_0 : \mu = 500$ where μ denotes $H_1 : \mu < 500$ the population mean α: $z = \frac{435-500}{100/\sqrt{4}} = -1.3$ Compare –1.282	B2 M1 A1 B1		Both hypotheses stated correctly [SR: 1 error, B1, but $\bar{x}$ etc: B0] Standardise, use $\sqrt{4}$, can be + $z = -1.3$ (allow –1.29 from cc) *or* $\Phi(z) = 0.0968$ (.0985) Compare *z* & –1.282 *or p* (< 0.5) & 0.1 or equivalent
		β: 500 – 1.282×100/√4 = 435.9; compare 435	M1 A1√;B1		500 – *z*×100/√4, allow √ errors, any Φ^{-1}, must be – CV correct, √ on their *z*; 1.282 correct and compare
		Reject H_0 Significant evidence that number of visitors has decreased	M1√ A1√	**7**	Correct deduction, needs √4, $\mu = 500$, like-with-like Correct conclusion interpreted in context
	(ii)	CLT doesn't apply as *n* is small So need to know distribution	M1 B1	**2**	Correct reason ["*n* is small" is sufficient] Refer to distribution, e.g. "if not normal, can't do it"
6	(i)	(a) 1 – 0.8153 = 0.1847 (b) 0.8153 – 0.6472 = **0.168**	M1 A1 M1 A1	**2** **2**	Po(3) tables, "1 –" used, e.g. 0.3528 or 0.0839 Answer 0.1847 or 0.185 Subtract 2 tabular values, or formula [$e^{-3}\ 3^4/4!$] Answer, a.r.t. 0.168
	(ii)	N(150, 150) $1 - \Phi\left(\frac{165.5-150}{\sqrt{150}}\right)$ = 1 – Φ(1.266) = **0.103**	B1 B1 M1 A1 A1	**5**	Normal, mean 3×50 stated or implied Variance or SD = 3 × 50, or same as μ Standardise 165 with λ, $\sqrt{\lambda}$ or λ, any or no cc $\sqrt{\lambda}$ and 165.5 Answer in range [0.102, 0.103]
	(iii)	(a) The sale of one house does not affect the sale of any others (b) The average number of houses sold in a given time interval is constant	B1 B1	**2**	Relevant answer that shows evidence of correct understanding [but *not* just examples] Different reason, in context [Allow "constant rate" or "uniform" but not "number constant", "random", "singly", "events".]

Final Mark Scheme (post-standardisation)

7	(i)	$\int_0^2 kx\,dx = \left[\frac{kx^2}{2}\right]_0^2 = 2k$ $= 1$ so $k = ½$	M1 A1 **2**	Use $\int_0^2 kx\,dx = 1$, or area of triangle Correctly obtain $k = ½$ **AG**
	(ii)		B1 B1 **2**	Straight line, positive gradient, through origin Correct, some evidence of truncation, no need for vertical
	(iii)	$\int_{\frac{1}{2}}^2 x^2\,dx = \left[\frac{1}{6}x^3\right]_0^2 = \frac{4}{3}$ $\int_{\frac{1}{2}}^2 x^3\,dx = \left[\frac{1}{8}x^4\right]_0^2$ [= 2] $2 - \left(\frac{4}{3}\right)^2 = \frac{2}{9}$	M1 A1 M1 M1 A1 **5**	Use $\int_0^2 kx^2\,dx$; $\frac{4}{3}$ seen or implied Use $\int_0^2 kx^3\,dx$; subtract their mean^2 Answer $\frac{2}{9}$ or a.r.t. 0.222, c.a.o.
	(iv)		M1 A1√ **2**	Translate horizontally, allow stated, or "1, 2" on axis One unit to right, 1 and 3 indicated, nothing wrong seen, no need for vertical or emphasised zero bits [If in doubt as to → or ↓, M0 in this part]
	(v)	$\frac{7}{3}$ $\frac{2}{9}$	B1√ B1√ **2**	Previous mean + 1 Previous variance [If in doubt as to → or ↓, B1B1 in this part]
8	(i)	$H_0: p = 0.65$ OR $p \geq 0.65$ $H_1: p < 0.65$ B(12, 0.65)	B2 M1	Both hypotheses correctly stated, in this form [One error (but not r, x or $\bar{x}$): B1] B(12, 0.65) stated or implied
		α: P(≤ 6) = 0.2127 Compare 0.10	A1 B1	Correct probability from tables, *not* P(= 6) Explicit comparison with 0.10
		β: Critical region ≤ 5; 6 > 5 Probability 0.0846	B1 A1	Critical region ≤5 or ≤6 or {≤4} ∩ {≥11} & compare 6 Correct probability
		Do not reject H_0 Insufficient evidence that proportion of population in favour is not at least 65%	M1√ A1√ **7**	Correct comparison and conclusion, needs correct distribution, correct tail, like-with-like Interpret in context, e.g. "consistent with claim" [SR: N(7.8, 2.73): can get B2M1A0B1M0: 4 ex 7]
	(ii)	Insufficient evidence to reject claim; test and p/q symmetric	B1√ B1 **2**	Same conclusion as for part (i), don't need context Valid relevant reason, e.g. "same as (i)"
	(iii)	$R \sim B(2n, 0.65)$, $P(R \leq n) > 0.15$ B(18, 0.65), $p = 0.1391$ Therefore $n = 9$	M1 A1 A1 A1 **4**	B(2n, 0.65), $P(R \leq n) > 0.15$ stated or implied Any probability in list below seen $p = 0.1391$ picked out (i.e., not just in a list of > 2) Final answer $n = 9$ only [SR <n: M1A0, $n = 4$, 0.1061 A1A0] [SR 2-tail: M1A1A0A1 for 15 or 14] [SR: 9 only, no working: M1A1] [MR B(12, 0.35): M1A0, $n = 4$, 0.1061 A1A0]

3	0.3529	7	0.1836	12	0.0942
4	0.2936	8	0.1594	13	0.0832
5	0.2485	9	0.1391	14	0.0736
6	0.2127	10	0.1218	15	0.0652